

# Momentum distributions in light halo nuclei and structure constraints

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**Abstract.** The core recoil momentum distribution of neutron-rich isotopes of light exotic nuclei is studied within a three-body model, where the nuclei are described by a core and two neutrons, with interactions dominated by the s-wave channel. In our framework, the two-body subsystems should have large scattering lengths in comparison with the interaction range allowing to use a three-body model with a zero-range force. The ground-state halo wave functions in momentum space are obtained by using as inputs the two-neutron separation energy and the energies of the singlet neutron-neutron and neutron-core virtual states. Within our model, we obtain the momentum probability densities for the Borromean exotic nuclei  $^{11}\text{Li}$  and  $^{22}\text{C}$ . In the case of the core recoil momentum distribution of  $^{11}\text{Li}$ , a fair reproduction of the experimental data was obtained, without free parameters, considering only the two-body low-energies. By analysing the obtained core momentum distribution in face of recent experimental data, we verify that such data are constraining the  $^{22}\text{C}$  two-neutron separation energy to a value between 100 and 400 keV.

## 1 Introduction

The observation of a large enhancement of the reaction cross sections of  $^{22}\text{C}$  on liquid hydrogen target at 40A MeV compared to the neighbour nuclei  $^{19}\text{C}$  and  $^{20}\text{C}$  by Tanaka and collaborators [1], analysed with a finite-range Glauber calculation under an optical-limit approximation suggested a matter root-mean-square (rms) radius of  $5.4 \pm 0.9$  fm. In addition, their analysis showed that the two-valence neutrons occupy preferentially the  $1s_{1/2}$  orbital. Such a large matter radius taken together with the  $^{20}\text{C}$  one indicates a rms radius of the halo neutron orbit of  $15 \pm 4$  fm estimated in [2]. The information on the rms matter radius was used to constrain the poorly known experimental value of the two-neutron separation energy of  $^{22}\text{C}$  ( $S_{2n}$ ) to be below 100 keV. This value was found consistently in calculations performed with a renormalized three-body model [2], with a shell model approach [3]

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and with effective field theory with a contact interaction [4]. However, the comparison between the experimental data for the core recoil momentum distributions of  $^{11}\text{Li}$  [5] and  $^{22}\text{C}$  [6] indicates similar sizes of their halos composed by two neutrons, which suggest that the matter radius of this carbon isotope could be overestimated (for further discussions see [7]).

This motivates us to study the properties of low-momentum distributions in two-neutron halo light exotic nucleus, which are dominated by s-wave short-range two-body interactions in the neutron-neutron and neutron-core subsystems. Our aim is to investigate theoretically within a three-body model, the core recoil momentum distribution as obtained experimentally by the halo breakup in nuclear targets. Our interest is to provide a constraint to the parameters associated with the halo structure and two-neutron separation energy based on these data fitted to three-body model calculations.

The model calculations for the core recoil momentum distribution are performed within a zero-range three-body model applied to a core-neutron-neutron system [9, 10]. The detailed formulation of the momentum distributions was given in [8] and the inputs of the zero-range model are the scattering lengths, and one three-body scale, which is given by the two-neutron separation energy. Therefore, three low-energy observables are enough to determine another low-energy quantity through an appropriate universal scaling function computed within the zero-range model. For example, the width of the momentum distribution is universally correlated to these physical scales and once the two-body scattering lengths are given, it is possible to constraint  $S_{2n}$  if the width is known experimentally.

The data for  $^{11}\text{Li}$  (see e.g. [5]) is used as a test of the reliability of our model analysis, as the low energy parameters used as input to our calculations are known. The recent experimental results for  $^{20}\text{C}$  and  $^{22}\text{C}$  [6] allow us, in principle, to constraint  $S_{2n}$  and the matter radius of  $^{22}\text{C}$ . In the preliminary analysis presented in this contribution, we focus on the constrain to  $S_{2n}$  obtained by computing the core recoil momentum distribution of  $^{22}\text{C}$  and fitting it to the experimental data from reference [6].

In sect. 2, we briefly sketch the zero-range three-body model and the core recoil momentum distribution formula, used in our analysis for the  $^{11}\text{Li}$  and  $^{22}\text{C}$  data. In sect. 3, we present our calculations for the momentum distributions compared to the data for these two cases, and provide a summary of our findings.

## 2 Model for the neutron-neutron-core system

The core recoil momentum distribution in the neutron-neutron-core (n-n-c) system is given by [8]:

$$n(q_c) = \int d^3 p_c |\langle \vec{q}_c \vec{p}_c | \Psi \rangle|^2, \quad (1)$$

where the relative momentum of the core to the nn subsystem is  $\vec{q}_c$  and the relative momentum between the neutrons is  $\vec{p}_c$ , which is integrated above. The renormalized zero-range three-body model for the two-neutron halo nuclei is detailed in Ref.[9]. The s-wave three-body wave function for the model ( $|\Psi\rangle$ ) can be written in terms of spectator functions,  $\chi_{nn}(q_c)$  and  $\chi_{nc}(|\vec{p}_c \pm \frac{\vec{q}_c}{2}|)$ , as

$$\langle \vec{q}_c \vec{p}_c | \Psi \rangle = \frac{\chi_{nn}(q_c) + \chi_{nc}(|\vec{p}_c - \frac{\vec{q}_c}{2}|) + \chi_{nc}(|\vec{p}_c + \frac{\vec{q}_c}{2}|)}{E_3 + H_0}, \quad (2)$$

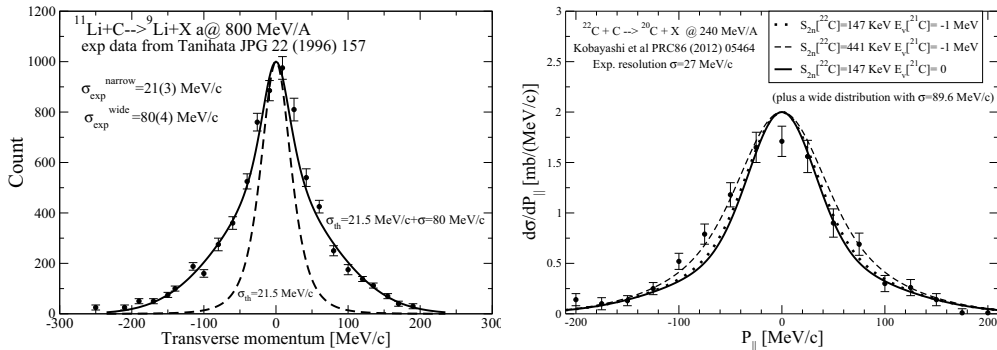
where the free Hamiltonian is

$$H_0 \equiv \frac{p_c^2}{2m_{nn}} + \frac{q_c^2}{2m_{nn,c}}. \quad (3)$$

Defining  $\mathcal{A} \equiv \frac{m_c}{m_n}$ , the reduced masses are given by  $m_{nn} = \frac{m_n}{2}$  and  $m_{nn,c} = \frac{2\mathcal{A}}{\mathcal{A}+2}m_n$ , where  $m_n$  is neutron mass. The core is assumed structureless and the two neutrons are in a singlet spin state to supply the correct antisymmetrization of the neutrons state in the halo.

The spectator functions come from the solution of a coupled set of integral equations corresponds to the Faddeev equations subtracted at a given scale (see [9]). The inputs are the nn and nc scattering lengths, or in the Borromean case the virtual state energies. In addition to these low energy parameters the value of the subtraction energy scale  $-\mu^2$  has to be provided. This is done by fixing  $S_{2n}$ . The value of  $\mu$  can be moved towards infinity and a tower of Thomas-Efimov states appears, with the shallowest one having its infrared properties fixed only by the three low energy quantities, the virtual energies of the subsystems and  $S_{2n}$ .

The log-periodic behaviour appear in the ultraviolet region not affecting low momentum part of the wave function that indeed runs to a limit cycle. This behaviour is explored in details in [8], where it was also derived an universal ration between the magnitude of the log-periodic spectator functions in the limit of  $\mu \rightarrow \infty$ , which is a function of only the mass ratio  $A$ . In practice the first cycle is enough to get the proper ties of the wave function in the infrared momentum region, which is acceptable for our purposes and used in our computations of the wave function. In the actual case of  $^{11}\text{Li}$  and  $^{22}\text{C}$  we have  $\mu^2 \gg S_{2n}$ , and therefore we are close to the universal limit cycle result in the low momentum region of the core recoil distribution of interest for the present study.



**Figure 1.** Core recoil momentum distribution for  $^{11}\text{Li}$  and  $^{22}\text{C}$ . *Left Frame:* The experimental data for  $^{11}\text{Li}$  from [5] is compared to the zero-range model calculation with no free parameters (see the explanation in the text). Theory with a narrow distribution (dashed line) with  $\sigma_{th} = 21.5$  MeV/c. Theory with a narrow distribution added to a wide one (solid line) with  $\sigma_{th} (=21.5$  MeV/c) +  $\sigma (=80$  MeV/c). The resulting distributions are fitted to the peak. *Right Frame:* The experimental data for  $^{22}\text{C}$  from [6] is compared to the zero-range model calculation with different input parameters. The results obtained for the virtual state energy of  $^{21}\text{C}$ ,  $E_V[^{21}\text{C}] = -1$  MeV, are shown for two values of  $S_{2n}$  given by 441 keV (dashed line) and 147 keV (dotted line). The calculations for  $E_V[^{21}\text{C}] = 0$  and  $S_{2n} = 147$  keV are given by the solid line. The computed narrow distribution is folded with the experimental resolution of  $\sigma = 27$  MeV/c. It is added to a wide one with width 89.6 MeV/c and fitted to the peak value of 2 mb/(MeV/c).

### 3 Results and Summary

We start the presentation of our results in figure 1 for  $^{11}\text{Li}$  (left frame), by checking the reliability of the three-body approach to compute the core distribution density for a weakly bound two-neutron halo nuclei. In this case the s-wave virtual states of  $^{10}\text{Li}$  and nn are fixed to  $E_V[^{10}\text{Li}] = -50$  keV and  $E_V[nn] = -147$  keV, respectively. The two-neutron separation energy of  $^{11}\text{Li}$  is given by the experimental value of 369 keV [11]. We compare our results with the experimental data for the core transverse momentum distribution obtained from the  $^{11}\text{Li}$  breakup on a carbon target at 800 MeV/A [5]. A wide distribution with  $\sigma = 80$  MeV/c is added to the computed narrow one with width  $\sigma = 21.5$  MeV/c, which should be compared with the experimental one of 21(3) MeV/c. The wide momentum

distribution is outside the description of our model. It should be associated with inner part of the halo neutron orbits close to the core region. We show in the figure the comparison with the experimental data, where the normalisations of the wide and narrow distributions are fitted to the data. After that we find a fair reproduction of the experimental momentum distribution as shown in the figure. This test indicates that our approach is a viable tool to extract information on the large two-neutron halo properties from the core momentum distribution.

We apply the neutron-neutron-core model to describe the halo of  $^{22}\text{C}$  and the results are shown in figure 1 (right frame). We show a sample of results for the core recoil momentum distribution in  $^{22}\text{C}$  compared with the data from [6] with given separation energies of 147 and 441 KeV. The virtual s-wave energy for the singlet nn state is fixed to  $E_V [nn] = -147$  keV. We allow a variation of the virtual state energy of  $^{21}\text{C}$  with values 0 and -1 MeV [12] to span different possibilities. We have added a wide momentum distribution to the narrow one as indicated in the figure. In addition the experimental resolution from [6] was folded with our theoretical results. We observe that the core recoil momentum distribution does not exhibit a strong sensitivity to the variation of  $E_V [^{21}\text{C}]$  with  $S_{2n} = 147$  keV. The comparison in the figure shows a difference between the distributions obtained with  $S_{2n} = 147$  and 441 keV, which however are within the error of the experimental data. This suggests that the previous works based only the matter radius from [1] underestimate the two neutron separation energy.

In summary, our model was tested in the case of the core recoil momentum distribution of  $^{11}\text{Li}$ , where we found a fair reproduction of the experimental data, using only the known low energy parameters. In view of that, we claim that the model can be useful to analyse experimental data on core momentum distribution of other halo nuclei. In particular, by analysing the recent experimental data on the core momentum distribution in  $^{22}\text{C}$ , given by Kobaiashi et al. [6], we are able to conclude that the value of the  $^{22}\text{C}$  two-neutron separation energy is within the interval of 100 and 400 keV.

## Acknowledgements

This work was partially supported by the Brazilian agencies CAPES, FAPESP and CNPq.

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